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Laplace transform method for scattering on Coulomb plus nonlocal separable potentials

D K Ghosh, M R Sinha and B Talukdar
Department of Physics, Visva-Bharati University, Santiniketan 731235, West Bengal, India

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Abstract. A Laplace transform method is derived to solve the Schrödinger equation for the Coulomb plus Tabakin potentials with the regular boundary condition. The results are used to construct expressions for on- and off-shell Jost functions in terms of Gaussian hypergeometric functions.

1. Introduction

The continuum solutions of the radial Schrödinger equation

$$\left(\frac{d^2}{dr^2}+k^2-\frac{l(l+1)}{r^2}-\frac{2\eta k}{r}\right)\phi_l(k,r)=\sum_{i=1}^{N}\lambda_i v_l^{(i)}(r)\int_0^\infty v_l^{(i)}(s)\phi_l(k,s)\,ds \tag{1.1}$$

for the Coulomb plus finite-rank separable nucleon–nucleon potential have important applications in charged particle scattering. For example, non-relativistic models for proton–proton (pp) scattering have a short-range potential built in to account for the strong interaction, the Coulomb potential taking care of the charges. The object of the present paper is to develop a Laplace transform method to solve equation (1.1) for the regular boundary condition (Newton 1966) and use this solution to construct expressions for on- and off-shell Jost functions (Jost 1947, Fuda 1976). In equation (1.1) the two-body centre of mass energy $E=k^2$, $2\eta k/r$ is the Coulomb potential, η being the well known Sommerfeld parameter, and $v_l^{(i)}(r)$ is the state-dependent form factor of the separable potential. The method proposed in this paper will work for arbitrary angular momentum and rank-N separable potentials. However, for clarity of presentation we specialise equation (1.1) to the s-wave case only and deal with (i) Coulomb plus rank-one and (ii) Coulomb plus rank-two Tabakin potentials (Tabakin 1965, 1968). For the sake of brevity we omit the subscript $l=0$ and work in units in which $\hbar^2/2m$ is unity. The reason for our interest in the Tabakin potentials is the following. The Tabakin potentials have been parametrised for the $^1S_0$ state. The s-wave pp scattering involves this state only, the triplet spin state being forbidden by the exclusion principle.

2. Regular solutions

Here we obtain the regular solution of equation (1.1).

(i) For the rank-one Tabakin form factor

$$v(r)=(A_1\cos\alpha_1 r+A_2\sin\alpha_1 r)\exp(-\alpha_1 r)+A_3\exp(-\alpha_2 r) \tag{2.1}$$

the integrodifferential equation (1.1) reads

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}r^2}+k^2-\frac{2\eta k}{r}\right)\phi(k,r)=d(k)[A\exp(-\beta r)+A^*\exp(-\beta^* r)+A_3\exp(-\alpha_2 r)], \tag{2.2}$$

where

$$d(k)=\lambda\int_0^\infty[A\exp(-\beta s)+A^*\exp(-\beta^* s)+A_3\exp(-\alpha_2 s)]\phi(k,s)\,\mathrm{d}s \tag{2.3}$$

with

$$A=\tfrac{1}{2}(A_1-\mathrm{i}A_2),\qquad \beta=\alpha_1-\mathrm{i}\alpha_1. \tag{2.4}$$

The asterisk stands for complex conjugation. We shall solve equation (2.2) by treating the integral (2.3) as a constant. The unknown constant which appears will be determined by substituting the solution back in the defining equation for $d(k)$ and matching the desired boundary conditions (Talukdar *et al* 1979, Talukdar and Das 1979). Using the transformations

$$\phi(k,r)=r\,\mathrm{e}^{\mathrm{i}kr}g(r),\qquad r=-z/2\mathrm{i}k, \tag{2.5}$$

in equation (2.2) we get

$$z\frac{\mathrm{d}^2g(z)}{\mathrm{d}z^2}+(c-z)\frac{\mathrm{d}g(z)}{\mathrm{d}z}-ag(z)=-\frac{d(k)}{2\mathrm{i}k}[A\exp(\rho_1 z)+A^*\exp(\rho_2 z)+A_3\exp(\rho_3 z)], \tag{2.6}$$

where

$$c=2,\qquad a=1+\mathrm{i}\eta,\qquad \rho_1=\frac{\beta+\mathrm{i}k}{2\mathrm{i}k},\qquad \rho_2=\frac{\beta^*+\mathrm{i}k}{2\mathrm{i}k},\qquad \rho_3=\frac{\alpha_2+\mathrm{i}k}{2\mathrm{i}k}. \tag{2.7}$$

Equation (2.6) represents a non-homogeneous linear differential equation (Babister 1967). Complementary functions of this equation are given by the confluent hypergeometric functions

$$\Phi(a,c;z)=\frac{\Gamma(c)}{\Gamma(a)}\sum_{n=0}^{\infty}\frac{\Gamma(a+n)z^n}{\Gamma(c+n)n!} \tag{2.8}$$

and

$$\bar{\Phi}(a,c;n)=z^{1-c}\Phi(a-c+1,2-c;z). \tag{2.9}$$

Clearly, for $c=2$, equation (2.9) is not an acceptable solution of (2.6). However, it tends towards the solution (Erdelyi 1953) of (2.6) when c approaches 2. In our subsequent discussion we shall always mean that limit. This is no loss of generalisation (Newton 1966).

Since confluent hypergeometric functions are of exponential order, and the RHS of (2.6) also is exponential, the Laplace transform method is expected to serve as one of the best techniques to solve this. If $\mathrm{Re}\, c<2$ both parts of the complementary functions have transforms; if $\mathrm{Re}\, c \geqslant 2$ only one part has. Taking the Laplace transform of (2.6) we get

$$\frac{\mathrm{d}}{\mathrm{d}s}\{s(1-s)\bar{g}(s)\}+(cs-a)\bar{g}(s)=(c-1)g(0)-\frac{d(k)}{2ik}\left(\frac{A}{s-\rho_1}+\frac{A^*}{s-\rho_2}+\frac{A_3}{s-\rho_3}\right), \tag{2.10}$$

where $\bar{g}(s)=\mathscr{L}[g(z)]$. This is a first-order differential equation in $\bar{g}(s)$ and can easily be solved to appear in the form

$$\begin{aligned}\bar{g}(s)=s^{a-1}(s-1)^{c-a-1}\bigg[R&+(c-1)g(0)\int_s^\infty \frac{\mathrm{d}\omega}{\omega^a(\omega-1)^{c-a}}\\ &-\frac{d(k)}{2ik}\left(A\int_s^\infty \frac{\mathrm{d}\omega}{\omega^a(\omega-1)^{c-a}(\omega-\rho_1)}+A^*\int_s^\infty \frac{\mathrm{d}\omega}{\omega^a(\omega-1)^{c-a}(\omega-\rho_2)}\right.\\ &\left.+A_3\int_s^\infty \frac{\mathrm{d}\omega}{\omega^a(\omega-1)^{c-a}(\omega-\rho_3)}\right)\bigg],\end{aligned} \tag{2.11}$$

where R is a constant. The first two terms on the RHS of (2.11) give the complementary functions of (2.6), while the last three terms give the particular integrals. This can be shown as follows.

Consider the standard integral

$$\int_0^\infty \mathrm{e}^{-sz}z^\nu\Phi(a,c;pz)\,\mathrm{d}z=\frac{\Gamma(\nu+1)}{s^{\nu+1}}\,{}_2F_1(a,\nu+1;c;p/s) \tag{2.12}$$

and the integral representation for the Gaussian hypergeometric function

$${}_2F_1(\alpha,\beta;\gamma;z)=\frac{\Gamma(\gamma)}{\Gamma(\beta)\Gamma(\gamma-\beta)}\int_0^1 t^{\beta-1}(1-t)^{\gamma-\beta-1}(1-tz)^{-\alpha}\,\mathrm{d}t. \tag{2.13}$$

Combining (2.12) and (2.13) and using the transformation $\omega=(s-t)/(1-t)$, it is easy to see that

$$(c-1)\mathscr{L}^{-1}\left(s^{a-1}(s-1)^{c-a-1}\int_s^\infty \frac{\mathrm{d}\omega}{\omega^a(\omega-1)^{c-a}}\right)=\Phi(a,c;z) \tag{2.14}$$

and

$$\mathscr{L}^{-1}[s^{a-1}(s-1)^{c-a-1}]=[\Gamma(2-c)]^{-1}\bar{\Phi}(a,c;z). \tag{2.15}$$

To deal with the last three terms in (2.11) we restrict ourselves to the half plane $\mathrm{Re}\, s>\mathrm{Re}\,\rho_i$ and $\mathrm{Re}\, s>1$, $i=1, 2, 3$. Thus

$$\int_s^\infty \frac{\mathrm{d}\omega}{\omega^a(\omega-1)^{c-a}(\omega-\rho_i)}=\sum_{n=0}^\infty \rho_i^n\int_s^\infty \frac{\mathrm{d}\omega}{\omega^{a+n+1}(\omega-1)^{c-a}}. \tag{2.16}$$

Allowing $a\to a+n+1$, $c\to c+n+1$ in (2.14) and using the series expansion of ${}_2F_1(\alpha,\beta;\gamma;z)$, we have

$$\mathscr{L}^{-1}\left(s^{a-1}(s-1)^{c-a-1}\int_s^\infty \frac{\mathrm{d}\omega}{\omega^{a+n+1}(\omega-1)^{c-a}}\right)=\frac{\theta_{n+1}(a,c;z)}{n!}, \tag{2.17}$$

where $\theta_\sigma(a, c; z)$ has been given in Babister (1967):

$$\theta_\sigma(a,c;z) = z^\sigma \sum_{m=0}^{\infty} \frac{\Gamma(\sigma+a+m)\Gamma(\sigma)\Gamma(\sigma+c-1)}{\Gamma(\sigma+a)\Gamma(\sigma+m+1)\Gamma(\sigma+c+m)} z^m. \tag{2.18}$$

In view of (2.14), (2.15) and (2.17) the inverse transform of (2.11) can be taken as

$$g(z) = \Phi(a,c;z) - \frac{d(k)}{2ik} \sum_{n=1}^{\infty} \frac{\theta_n(a,c;z)}{(n-1)!}(A\rho_1^{n-1} + A^*\rho_2^{n-1} + A_3\rho_3^{n-1}). \tag{2.19}$$

Note that for the regular boundary condition $R=0$ and $g(0)=1$. Combining (2.3), (2.5) and (2.18) we have

$$d(k) = \frac{\lambda}{D_1(k)}\left[\frac{A}{\beta^2+k^2}\left(\frac{\beta-ik}{\beta+ik}\right)^{i\eta} + \frac{A^*}{\beta^{*2}+k^2}\left(\frac{\beta^*-ik}{\beta^*+ik}\right)^{i\eta} + \frac{A_3}{\alpha_2^2+k^2}\left(\frac{\alpha_2-ik}{\alpha_2+ik}\right)^{i\eta}\right], \tag{2.20}$$

where the Fredholm determinant associated with the regular solution for the Coulomb plus rank-one Tabakin potential is

$$\begin{aligned} D_1(k) = 1 + \lambda \sum_{n=1}^{\infty} (-1)^n [A(\beta+ik)^{n-1} + A^*(\beta^*+ik)^{n-1} + A_3(\alpha_2+ik)^{n-1}] \\ \times\left[\frac{A}{(\beta-ik)^{n+2}} {}_2F_1\left(1+i\eta+n, 1; n+1; \frac{-2ik}{\beta-ik}\right)\right. \\ + \frac{A^*}{(\beta^*-ik)^{n+2}} {}_2F_1\left(1+i\eta+n, 1; n+1; \frac{-2ik}{\beta^*-ik}\right) \\ \left. + \frac{A_3}{(\alpha_2-ik)^{n+2}} {}_2F_1\left(1+i\eta+n, 1; n+1; \frac{-2ik}{\alpha_2-ik}\right)\right]. \end{aligned} \tag{2.21}$$

Thus the regular solution

$$\begin{aligned} \phi(k,r) = r\,e^{ikr}\Phi(1+i\eta, 2; -2ikr) - \frac{\lambda}{2ikD_1(k)}\left[\frac{A}{\beta^2+k^2}\left(\frac{\beta-ik}{\beta+ik}\right)^{i\eta}\right. \\ \left. + \frac{A^*}{\beta^{*2}+k^2}\left(\frac{\beta^*-ik}{\beta^*+ik}\right)^{i\eta} + \frac{A_3}{\alpha_2^2+k^2}\left(\frac{\alpha_2-ik}{\alpha_2+ik}\right)^{i\eta}\right] \\ \times r\,e^{ikr} \sum_{n=1}^{\infty} \frac{\theta_n(1+i\eta, 2; -2ikr)}{(n-1)!}(A\rho_1^{n-1} + A^*\rho_2^{n-1} + A_3\rho_3^{n-1}). \end{aligned} \tag{2.22}$$

A couple of useful checks can be made on the fairly complicated expression (2.22). For example, in the absence of the nuclear potential ($\lambda=0$) equation (2.22) gives the solution for the pure Coulomb field (Newton 1966). In the absence of the Coulomb field ($\eta=0$) we have from (2.22)

$$\begin{aligned} \phi(k,r) = r\,e^{ikr}(1, 2; -2ikr) - \frac{\lambda}{D_1(k)2ik}\left(\frac{A}{\beta^2+k^2} + \frac{A^*}{\beta^{*2}+k^2} + \frac{A_3}{\alpha_2^2+k^2}\right) \\ \times r\,e^{ikr} \sum_{n=1}^{\infty} \frac{\theta_n(1, 2; -2ikr)}{(n-1)!}(A\rho_1^{n-1} + A^*\rho_2^{n-1} + A_3\rho_3^{n-1}). \end{aligned} \tag{2.23}$$

Using the relations (Babister 1967)

$$\Phi(1, 2; z) = e^{z/2} i_0(z/2), \tag{2.24}$$

$$\sum_{m=1}^{\infty} \theta_m(1, 2; z)\frac{\rho^{m-1}}{(m-1)!} = \sum_{m=1}^{\infty} \frac{\rho^{m-1} z^m}{(m+1)!}\Phi(1, m+2; z), \tag{2.25}$$

together with the integral representation

$$\Phi(a, c; z) = \frac{\Gamma(c)}{\Gamma(a)\Gamma(c-a)}\int_0^1 e^{zt} t^{a-1}(1-t)^{c-a-1}\, dt, \tag{2.26}$$

it can be seen that $\phi(k, r)$ in (2.23) is in exact agreement with that given by Bagchi *et al* (1977).

(ii) For the rank-two Tabakin potential

$$V(r, r') = -g(r)g(r') + h(r)h(r') \tag{2.27}$$

with

$$g(r) = \gamma\, e^{-\alpha r}, \tag{2.28}$$

$$h(r) = \beta\, e^{-br}\{[(d^2 - b^2)/2db] \sin dr + \cos dr\}, \tag{2.29}$$

the integrodifferential equation (1.1) reads

$$\left(\frac{d^2}{dr^2} + k^2 - \frac{2\eta k}{r}\right)\phi(k, r) = d_1(k)\beta[B \exp(-ar) + B^* \exp(-a^* r)] - d_2(k)\gamma\, e^{-\alpha r}, \tag{2.30}$$

where

$$d_1(k) = \beta\int_0^{\infty} [B \exp(-as) + B^* \exp(-a^* s)]\phi(k, s)\, ds, \tag{2.31}$$

$$d_2(k) = \gamma\int_0^{\infty} e^{-\alpha s}\phi(k, s)\, ds, \tag{2.32}$$

with

$$B = \tfrac{1}{2} - i(d^2 - b^2)/2db, \qquad a = b - id. \tag{2.33}$$

Following the same procedure as before, one can obtain the regular solution for the Coulomb plus rank-two Tabakin potential

$$\phi(k, r) = r\, e^{ikr}(1 + i\eta, 2; -2ikr) - (2ik)^{-1} r\, e^{ikr} \times \sum_{n=1}^{\infty} \frac{\theta_n(1 + i\eta, 2; -2ikr)}{(n-1)!}[d_1(k)\beta(B\xi_1^{n-1} + B^*\xi_2^{n-1}) - d_2(k)\gamma\xi_3^{n-1}] \tag{2.34}$$

with

$$\xi_1 = \frac{a + ik}{2ik}, \qquad \xi_2 = \frac{a^* + ik}{2ik}, \qquad \xi_3 = \frac{\alpha + ik}{2ik}. \tag{2.35}$$

Using (2.34) in (2.31) and (2.32), and by solving the simultaneous equations for $d_1(k)$ and $d_2(k)$, we get

$$d_1(k) = \frac{\beta}{D_2(k)}\left\{[1 + \gamma^2 X_n(\alpha)]\left[\frac{B}{a^2 + k^2}\left(\frac{a - ik}{a + ik}\right)^{i\eta} + \frac{B^*}{a^{*2} + k^2}\left(\frac{a^* - ik}{a^* + ik}\right)^{i\eta}\right] - \frac{\gamma^2}{\alpha^2 + k^2}\left(\frac{a - ik}{\alpha + ik}\right)^{i\eta}[BY_n(\alpha, a) + B^* Y_n(\alpha, a^*)]\right\}, \tag{2.36}$$

$$d_2(k)=\frac{\gamma}{D_2(k)}\left\{\frac{1}{\alpha^2+k^2}\left(\frac{\alpha-\mathrm{i}k}{\alpha+\mathrm{i}k}\right)^{\mathrm{i}\eta}\{1-\beta^2B^2X_n(a)-\beta^2B^{*2}X_n(a^*)\right.$$
$$-\beta^2BB^*[Y_n(a,a^*)+Y_n(a^*,a)]\}$$
$$+\beta^2\left[\frac{B}{a^2+k^2}\left(\frac{a-\mathrm{i}k}{a+\mathrm{i}k}\right)^{\mathrm{i}\eta}+\frac{B^*}{a^{*2}+k^2}\left(\frac{a^*-\mathrm{i}k}{a^*+\mathrm{i}k}\right)^{\mathrm{i}\eta}\right]$$
$$\left.\times[BY_n(a,\alpha)+B^*Y_n(a^*,\alpha)]\right\}, \tag{2.37}$$

where the Fredholm determinant $D_2(k)$ associated with the regular solution for the Coulomb plus rank-two Tabakin potential is given by

$$D_2(k)=[1+\gamma^2X_n(\alpha)]\{1-\beta^2B^2X_n(a)-\beta^2B^{*2}X_n(a^*)$$
$$-\beta^2BB^*[Y_n(a,a^*)+Y_n(a^*,a)]\}$$
$$+\gamma^2\beta^2\{BY_n(a,\alpha)+B^*Y_n(a^*,\alpha)\}[BY_n(\alpha,a)+B^*Y_n(\alpha,a^*)] \tag{2.38}$$

with

$$Y_n(x,y)=\sum_{n=0}^{\infty}(-1)^n(x+\mathrm{i}k)^n\frac{1}{(y-\mathrm{i}k)^{n+3}}{}_2F_1\left(1,2+\mathrm{i}\eta+n;n+2;\frac{-2\mathrm{i}k}{y-\mathrm{i}k}\right), \tag{2.39}$$

$$X_n(x)=Y_n(x,x). \tag{2.40}$$

Checks similar to those in (i) can also be made for the rank-two potential treated above.

3. Jost functions

In terms of the regular solution of the Schrödinger equation, the s-wave on-shell (Newton 1966, de Alfaro and Regge 1965, Arnold and Seyler 1973) and off-shell (Fuda 1976) Jost solutions for the Coulomb plus rank-N separable potentials are given by

$$f(k)=f^{\mathrm{c}}(k)+\sum_{i=1}^{N}\lambda_i\int_0^\infty v^{(i)}(r)f^{\mathrm{c}}(k,r)\,\mathrm{d}r\int_0^\infty v^{(i)}(s)\phi(k,s)\,\mathrm{d}s, \tag{3.1}$$

$$f(k,q)=1+\int_0^\infty \mathrm{e}^{\mathrm{i}qr}\left(\frac{2\eta k}{r}\phi(k,r)+\sum_{i=1}^{N}\lambda_i v^{(i)}(r)\int_0^\infty v^{(i)}(s)\phi(k,s)\,\mathrm{d}s\right)\mathrm{d}r. \tag{3.2}$$

Here q is an off-shell momentum. Note that for Coulomb and Coulomb-like potentials (van Haeringen 1979)

$$\lim_{q\to k}f(k,q)\neq f(k). \tag{3.3}$$

In the above the Coulomb Jost solution and Jost function are

$$f^{\mathrm{c}}(k,r)=(-2\mathrm{i}k)\,\mathrm{e}^{\pi\eta/2}r\,\mathrm{e}^{\mathrm{i}kr}\Psi(1+\mathrm{i}\eta,2;-2\mathrm{i}kr), \tag{3.4}$$

$$f^{\mathrm{c}}(k)=\mathrm{e}^{\pi\eta/2}/\Gamma(1+\mathrm{i}\eta), \tag{3.5}$$

with $\Psi(a,c;z)$ an irregular confluent hypergeometric function (Erdelyi 1953).

(i) For the Coulomb plus rank-one Tabakin potential the on- and off-shell Jost functions obtained from (2.22), (3.1) and (3.2) are

$$f(k)=f^{c}(k)-2ik\,e^{\pi\eta/2}d(k)[AI(\beta)+A^{*}I(\beta^{*})+A_{3}I(\alpha_{2})], \tag{3.6}$$

$$\begin{aligned}f(k,q)=&f^{c}(k,q)+d(k)\left(\frac{A}{\beta-iq}+\frac{A^{*}}{\beta^{*}-iq}+\frac{A_{3}}{\alpha_{2}-iq}\right)\\&-2\eta kd(k)\sum_{n=1}^{\infty}\frac{i^{1-n}}{n+1}[A(\beta+ik)^{n-1}+A^{*}(\beta^{*}+ik)^{n-1}+A_{3}(\alpha_{2}+ik)^{n-1}]\\&\times{}_{2}F_{1}\left(1+i\eta+n,1;n+2;\frac{2k}{k+q}\right)(k+q)^{-n-1},\end{aligned} \tag{3.7}$$

where the Coulomb off-shell Jost function

$$f^{c}(k,q)=[(k+q)/(q-k)]^{i\eta} \tag{3.8}$$

and

$$\begin{aligned}I(x)=&\int_{0}^{\infty}r\,e^{-xr}\,e^{ikr}\Psi(1+i\eta,2;-2ikr)\,dr\\=&-\frac{1}{2ik\Gamma(1+i\eta)(x^{2}+k^{2})}\\&\times\left[x+2\eta k\,e^{2\eta y}\left(-\frac{i\pi}{2}+\frac{i}{2\eta}+\psi(1+i\eta)-\psi(1)+\ln\left(\frac{2k}{x}\right)\right.\right.\\&\left.\left.-\tfrac{1}{2}\ln\left(1+\frac{k^{2}}{x^{2}}\right)+\sum_{p=1}^{\infty}\frac{(-2\eta y)^{p}}{p!}\sum_{l=0}^{\infty}(-1)^{l}\frac{2^{2l}B_{2l}y^{2l}}{(2l)!(2l+p)}\right)\right]\end{aligned} \tag{3.9}$$

with $y=\tan^{-1}k/x$, B_{2l} the Bernoulli numbers and ψ the logarithmic derivative of the gamma function. The result in (3.9) can be obtained by following the method given by Talukdar *et al* (1982). In order to transform the first integral on the RHS in (3.2) to more useful forms, it is necessary at this point to introduce explicitly into the integral a convergence factor e^{-sr}. For all values of s such that Re $s\to 0$, the resulting integral is then uniformly convergent. The limit $s\to 0$ is taken in the final answer. Also in deriving (3.7) we have used the following integral (Babister 1967):

$$\begin{aligned}&\int_{0}^{\infty}e^{-\lambda z}z^{v}\theta_{\sigma}(\alpha,c;pz)\,dz\\&\quad=\frac{\Gamma(v+\sigma+1)}{\sigma(\sigma+c-1)}\frac{p^{\sigma}}{\lambda^{v+\sigma+1}}{}_{3}F_{2}(1,\sigma+\alpha,v+\sigma+1;\sigma+1,\sigma+c;p/\lambda)\end{aligned} \tag{3.10}$$

together with the reduction formula (Luke 1969)

$${}_{p}F_{q}(\alpha_{1},\beta_{1},\gamma_{1},\ldots;\alpha_{2},\beta_{1},\gamma_{2},\ldots;z)={}_{p-1}F_{q-1}(\alpha_{1},\gamma_{1},\ldots;\alpha_{2},\gamma_{2},\ldots;z). \tag{3.11}$$

(ii) Similarly for the Coulomb plus rank-two Tabakin potential, the on- and off-shell Jost functions obtained from (2.34), (3.1) and (3.2) are

$$f(k)=f^{c}(k)-2ik\,e^{\pi\eta/2}\{\beta d_{1}(k)[BI(a)+B^{*}I(a^{*})]-\gamma d_{2}(k)I(\alpha)\} \tag{3.12}$$

and

$$f(k,q)=f^{c}(k,q)+\beta d_1(k)\left(\frac{B}{a-\mathrm{i}q}+\frac{B^*}{a^*-\mathrm{i}q}\right)-d_2(k)\frac{\gamma}{\alpha-\mathrm{i}q}$$

$$-2\eta k\sum_{n=1}^{\infty}\frac{\mathrm{i}^{1-n}}{n+1}\frac{1}{(k+q)^{n-1}}\{d_1(k)\beta[B(a+\mathrm{i}k)^{n-1}+B^*(a^*+\mathrm{i}k)^{n-1}]$$

$$-d_2(k)\gamma(\alpha+\mathrm{i}k)^{n-1}\}{}_2F_1\left(1+\mathrm{i}\eta+n,1;n+2;\frac{2k}{k+q}\right). \tag{3.13}$$

The Jost functions for the Coulomb plus Yamaguchi potential (Yamaguchi 1954) can be obtained from (3.6) and (3.7) as well as from (3.12) and (3.13) in the limits $A_1, A_2 \to 0$ and $\beta \to 0$ respectively.

Some comments on our results for the Coulomb plus rank-one Tabakin potential are now in order. The rank-one Tabakin potential supports a positive energy bound state in the absence of the Coulomb potential. For such a bound state the S matrix $(=f(-k)/f(k))$ has a pole on the physical sheet. With the Coulomb potential turned on, this pole will move from the physical to the unphysical sheet. The positive energy bound state will be converted to a resonance. A detailed investigation of this point will be quite interesting.

4. Discussion

Experiments which involve scattering by additive interactions are analysed by the use of the Gell-Mann–Goldberger scattering-by-two-potentials theorem (GG theorem) (Gell-Mann and Goldberger 1953). Applicability of the GG theorem is directly related to the existence and/or completeness of the wave operators for the scattering system (Bajzer 1974). The wave operators exist under strong limits when each of the associated interactions is of short range, but they do not exist in the presence of a Coulomb force. To deal with long-range interactions, the wave operators are judiciously modified by relaxing some requirements. Recently, the situation with regard to this has been nicely summarised by Chandler (1981).

In this paper we have solved the Schrödinger equation for the Coulomb plus Tabakin potentials for the regular boundary condition without using the GG theorem and used these results to construct analytical expressions for the on- and off-shell Jost functions. The hypergeometric functions which occur here can be generated by using a three-term recurrence relation given in Snow (1952).

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